Solitary waves in one-dimensional heterogeneous granular media

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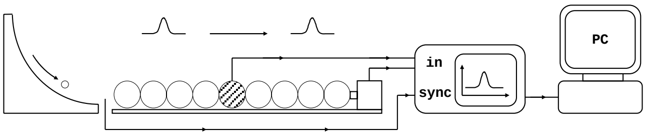
Experimental study of impulse propagation in 1D heterogeneous and unconstrained granular media.



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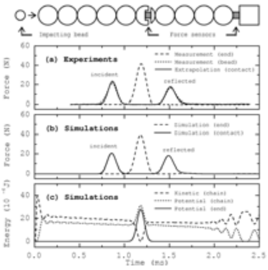
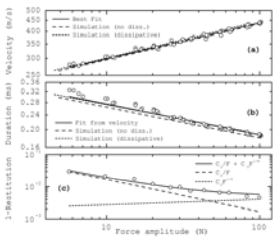
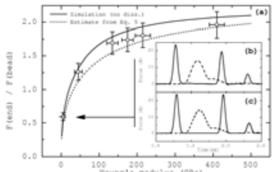
• We have designed, for this first experiment, a non-intrusive force sensor, which allows to measure the strain applied by an impulse that propagates in a chain of identical beads (a monodisperse chain), in the absence of static strain (punctual contacts between grains).

• We have obtained precise quantitative agreements between experiments, numerical simulations and analytical theory, and we have confirmed that an impulse propagates in such a medium in the form of a solitary wave with compact support.

• We have also shown a complex nonlinear reflection process, when a solitary wave reach a rigid and fixed extremity. The analysis of the reflection allows to determine the elastic properties of sample clamped at the end of the chain (probe of the material within a thickness of ten or so micrometers).







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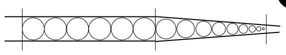
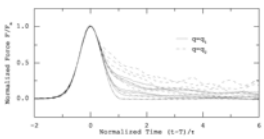
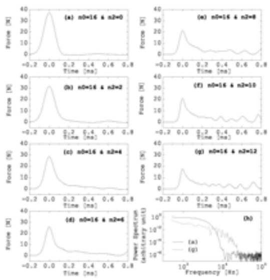
• We have studied here a granular medium made of beads whose diameters decrease exponentially (namely a tapered chain), without static strain.

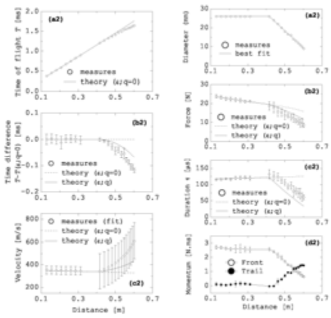
• The propagating impulse is relatively similar to the one that would propagate in a monodisperse chain, and follow same scaling laws. We have shown that the front of the impulse adopts a self-similar form. However, a train of wave is formed behind the front wave. The impulse disperses more and more, as it propagates far in the tapered chain.

• In comparison to a monodisperse chain, an additional dispersion process breaks the spatial compactness of the solitary wave solution, and modifications of the nonlinear terms tighten the impulse. This process allows to transfer a part of the energy and of the momentum of an impulse behind the front wave, and consequently diminishes the maximal strain reaching the end of the chain. The spectral density of the wave is also widen, toward higher frequencies.









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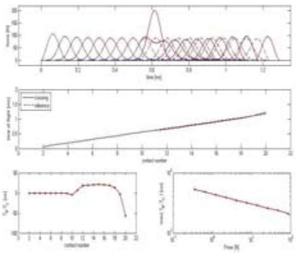

•We have designed an experimental setup for creating to solitary waves at both ends of the chain by using two non-intrusive force sensors. This setup allows us to measure both solitary waves simultaneously. We can also compare our experimental results with simulations (simulations presented in figure).

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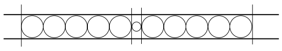

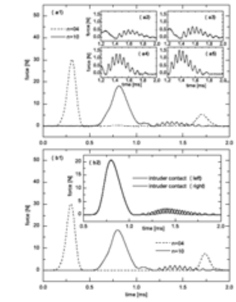

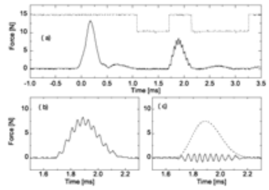

This configuration corresponds to a monodisperse chain of beads containing a nall intruder. Note that measurements realized with an intruder of bigger size dicates a different behavior.

When a solitary wave, or an impulse, reaches the intruder, we observe a patial localization of a part of the incident energy: the intruder starts to scillates, and the oscillations remains localized at the intruder location.

A dimensional analysis allowed us to correlates quantitatively the oscillations equency to the characteristic of the chain and of the incident wave.

It seems that localization of the energy is strongly related to the opening of a ap between the intruder and its nearest neighbors.

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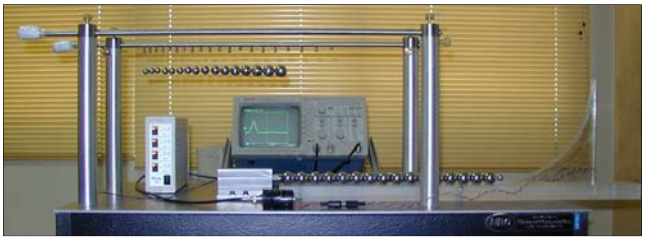


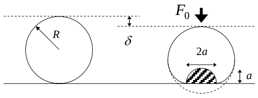
Elements of theory

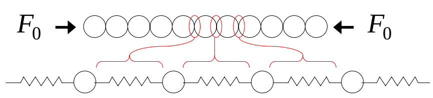
★ Hertz Contact: linear elasticity but nonlinear deformation of spherical contact

$$a^2 = R^2 - (R-\delta)^2 \approx 2R\delta \quad \Rightarrow \quad F_0 \sim ES(\delta/a) \sim E\delta a \sim E\delta^{3/2}$$

★ A chain of identical beads is a nonlinear dispersive media

$$k_{NL} = \partial F_0 / \partial \delta \propto F_0^{1/3} \quad \Rightarrow \quad \omega(q) = 2\sqrt{k_{NL}/m}\,|\sin(qR)|$$





★ Nonlinear and dispersive propagation equation (unconstrained monodisperse chain)

Newton's law: $m\partial_{tt}u_n = \kappa(u_{n-1}-u_n)^{3/2} - \kappa(u_n - u_{n+1})^{3/2}$ deformation: $\psi = -\partial_x u$ long wavelength approximation

$$\partial_{tt}\psi = c^2 \partial_{xx}\left[\psi^{3/2} + (2/5)R^2\psi^{1/4}\partial_{xx}\left(\psi^{5/4}\right)\right]$$

Elements of bibliography

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