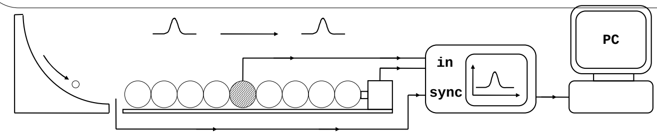
Solitary waves in one-dimensional heterogeneous granular media

Romina Muñoz, Francisco Santibañez, Stéphane Job[1], Francisco Melo[2]

(1) SUPMECA, 3 rue Fernand Hainaut, 93407 Saint-Ouen Cedex, France. www.supmeca.fr/perso/jobs

(2) Laboratorio de Fisica Nolineal, Universidad de Santiago de Chile (USACH)
Centro para la Investigación Interdisciplinaria Avanzada en Ciencias de los Materiales (CIMAT)
Avenida Ecuador 3493, Casilla 307, Correo 2 , Santiago de Chile.

Experimental study of impulse propagation in 1D heterogeneous and unconstrained granular media.

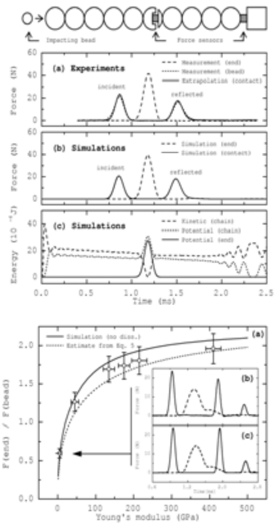


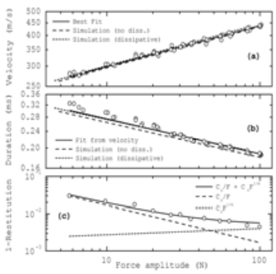
❶

• We have designed, for this first experiment, a non-intrusive force sensor, which allows to measure the strain applied by an impulse that propagates in a chain of identical beads (a monodisperse chain), in the absence of static strain (punctual contacts between grains).

• We have obtained precise quantitative agreements between experiments, numerical simulations and analytical theory, and we have confirmed that an impulse propagates in such a medium in the form of a solitary wave with compact support.

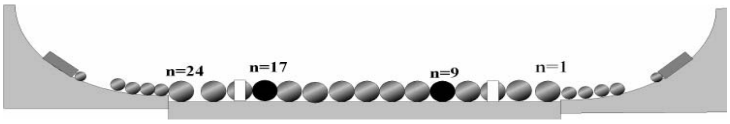
• We have also shown a complex nonlinear reflection process, when a solitary wave reach a rigid and fixed extremity. The analysis of the reflection allows to determine the elastic properties of sample clamped at the end of the chain (probe of the material within a thickness of ten or so micrometers).





Job, Melo, Sokolow et Sen

Crossing of two solitary waves in 1D granular chain with free boundary conditions.



❷

• Here we have studied the interaction between two solitary waves that propagates in a one dimensional chain. The highly non-linear behavior of this system is shown when the linear superposition assumption is invalid and the interaction of two solitary waves cannot be considered as the sum of two linear waves. Thus non-linearity generates a time delay in the solitary wave propagation time when it interacts with another solitary wave. This time delay is measured by means of in-situ force and time of flight measurements, done with two piezoelectric force sensors. (n=9, 17). Then, a time reference (tr) is measured for a single solitary wave propagating in the chain.

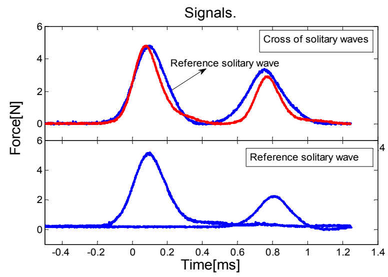
•We have studied three different cases:

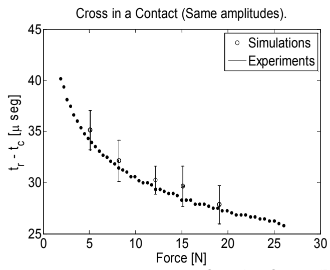
2.1) First, when the two waves have the same amplitude and the solitary waves cross in the contact between two beads.

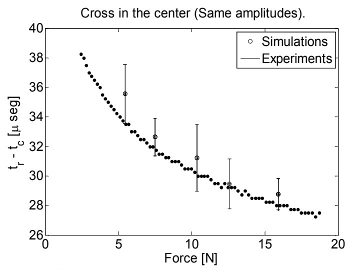
•2.2) When the two waves have the same amplitude and the solitary waves cross in the center of a bead.

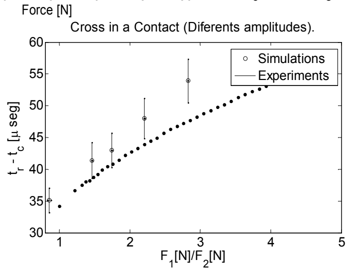
•2.3) And last when two waves have different amplitudes and the solitary waves cross in the contact between two beads.

The experimental results show the interaction between two solitary waves, the amount tr-tc increase when the amplitude of the waves decrease in cases (2.1) and (2.2) and tr-tc increase when the rate between the two amplitudes increase. This experimental results agree with numerical calculations.





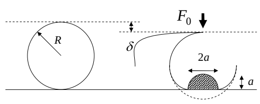




Elements of theory

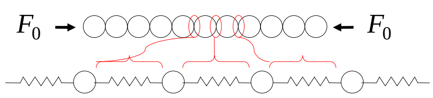
★ Hertz Contact: linear elasticity but nonlinear deformation of spherical contact

$$a^2 = R^2 - (R-\delta)^2 \approx 2R\delta \quad \Rightarrow \quad F_0 \sim ES(\delta/a) \sim E\delta a \sim E\delta^{3/2}$$



★ A chain of identical beads is a nonlinear dispersive media

$$k_{NL} = \partial F_0/\partial\delta \propto F_0^{1/3} \quad \Rightarrow \quad \omega(q) = 2\sqrt{k_{NL}/m}\,|\sin(qR)|$$



★ Nonlinear and dispersive propagation equation (unconstrained monodisperse chain)

Newton's law: $m\partial_{tt}u_n = \kappa(u_{n-1}-u_n)^{3/2} - \kappa(u_n - u_{n+1})^{3/2}$ deformation: $\psi = -\partial_x u$ long wavelength approximation

$$\partial_{tt}\psi = c^2\partial_{xx}\left[\psi^{3/2} + (2/5)R^2\psi^{1/4}\partial_{xx}\left(\psi^{5/4}\right)\right]$$

Elements of bibliography

- Landau et Lifchitz, *Théorie de l'élasticité*, Ed. Mir, Moscou (1967)
- Nesterenko, *Dynamics of heterogeneous materials*, Springer-Verlag, New York (2001)
- Coste, Falcon et Fauve, Solitary Waves in a chain of beads under Hertz contact, *Phys. Rev. E* 56, 6104-6117 (1997)
- Peyrard et Dauxois, *Physique des solitons*, EDP Sciences/CNRS Editions (2004)
- Job, Melo, Sokolow et Sen, *How Hertzian Solitary Waves Interact with Boundaries in a 1D Granular Medium*, Phys. Rev. Lett. 94, 178002 (2005)
- Melo, Job, Santibanez et Tapia, *Experimental evidence of shock mitigation in a Hertzian tapered chain*, Phys. Rev. E, 73, 041305 (2006)
- Santibanez, Tapia, Melo et Job, *Evidence of spatially localized excitations in Hertzian chains containing a small intruder* (en préparation)

DygraM2006 – Université de Rennes (June 19-23, 2006)